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Modeling natural convection with the work of pressure-forces: a thermodynamic necessity

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Abstract

Purpose – This paper aims to present and then resolve the thermodynamic inconsistencies inherent in the usual Boussinesq model, especially with respect to the second law, and to highlight the effects of the correction.

Design/methodology/approach – The Boussinesq model (i.e. still assuming $\nabla v = 0$) is made thermodynamically consistent by maintaining in the heat equation, primarily the work of pressure forces, secondarily the heat generated by viscous friction. Numerically speaking, the modifications are very easy and hardly affect the computing time. However, new non-dimensional parameters arise, especially the non-dimensional adiabatic temperature gradient, ϕ.

Findings – There are presented and interpreted results of systematic numerical simulations done for a two-dimensional square differentially-heated cavity filled with air at 300K, with Rayleigh number ranging from 3,000 to 10^8 and ϕ ranging from 10^{-3} to 2. All configurations are stationary and the fluid is far from its critical state. Nevertheless, the pressure-work effect (similar to the piston effect) enhances the heat transfer while diminishing the convection intensity. The magnitude of this effect is non-negligible as soon as ϕ reaches 0.02.

Practical implications – The domain where the thermodynamic Boussinesq model must be used encompasses configurations relevant to building engineering.

Originality/value – Exact second-law analyses can be developed with the so-corrected model.

Keywords Thermodynamics, Convection, Heat transfer, Numerical control, Modelling

Paper type Research paper

Nomenclature

A_r = aspect ratio of the cavity, H/L
c_p = heat capacity ($\text{Jkg}^{-1}\text{K}^{-1}$)
g = gravity (ms^{-2})
H = height of the cavity (m)
L = width of the cavity (m)
N_I = number of irreversibility
Nu = Nusselt number
N_W = non-dimensional work rate
P = pressure (Pa)
Pr = Prandtl number
Q = heat-rate (W)
$\mathbf{q}$ = heat-flux-density vector, (Wm^{-2})
q_v = heat-rate locally generated by viscous friction (Wm^{-3})
Ra = Rayleigh number
s = specific entropy (Jkg^{-1})
t = time (s)
T = temperature (K)

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u^* = specific internal energy (Jkg^{-1})
u = non-dimensional horizontal component of velocity
v = specific volume, $1/\rho$ (m^3kg^{-1})
$\mathbf{v}$ = velocity vector, (ms^{-1})
V_z = vertical component of fluid velocity (ms^{-1})
w = non-dimensional vertical component of velocity
x = non-dimensional horizontal position
x_c = position of the cold wall, $= A_r^{-1}$
z = non-dimensional vertical position

Greek symbols
α = thermal diffusivity (m^2s^{-1})
β = isobaric expansion coefficient (K^{-1})
ΔT = temperature difference $T_h - T_c$, (K)
ϕ = non-dimensional adiabatic temperature gradient
Φ = heat-rate locally dissipated by friction
θ = non-dimensional temperature
ρ = density (m^3kg^{-1})
σ = local rate of entropy production ($WK^{-1}m^{-3}$)
Σ = total rate of entropy production (WK^{-1})
τ = non-dimensional time

Subscripts
0 = reference state
c = cold side
C = Carnot
h = hot side
m = mechanical energy
q = conductive (heat diffusion)
v = viscous
λ = purely conductive system

Introduction

More than a century ago, Oberbeck (1879) and Boussinesq (1903) established the famous Oberbeck-Boussinesq equations, which have been very helpful since then for modeling buoyancy-driven natural convection, see for instance Bejan (1984) or Gebhart *et al.* (1988) among numerous authors. As long as those equations are used for simulations dedicated to comparison with experimental data or comprehensive studies like the benchmark of De Vahl Davis (1983), they surely are pertinent. However, the current studies about natural convection are more and more refined and theoretical, involving second law analyses, stability analyses, or multiple solutions, so that one may wonder whether those equations are still adapted to the intended purposes. Several authors (Tritton, 1988; Gray and Giorgini, 1976; Velarde and Perez-Cordon, 1976) showed that the Boussinesq approximation is valid as long as the temperature difference is sufficiently limited for the fluid density (and the other thermophysical properties) to be assumed as uniform and constant. When this condition is not fulfilled, the problem is said non-Boussinesq; such problems are investigated since the with low-Mach-number models, e.g. by Paolucci (1982) or more recently Vierendeels *et al.* (2001). The present concern is completely different. It originates in the difference between two entropy balances, that of real natural convection in steady-state, and that of the system simulated with the usual Boussinesq (UB) equations. Indeed, the two systems (real and UB ones) do not have same entropy balances. The consequences of this fundamental thermodynamic inconsistency become visible when the temperature difference is very small, i.e. rather close to thermodynamic equilibrium (Pons and Le Quéré 2004, 2005a, b). The present study describes that thermodynamic inconsistency and investigates its consequences.

Natural convection and entropy balance

Non-dimensional quantities are well-known for energies, e.g. the *Nu* number is the ratio of the effective heat flux in steady-state or in average and that transferred by the purely conductive system (fluid at rest): $Nu = Q/Q_\lambda$. The same purely conductive system, more exactly its entropy production Σ_λ, can also be the reference for entropy balances, thus yielding non-dimensional entropy-productions or changes. This non-dimensionalization straightforwardly leads to an equality that must exist in steady-state (or in average) between the number of total irreversibility ($N_\mathrm{I} = \Sigma/\Sigma_\lambda$) and the Nu number, as demonstrated here under:

$$N_\mathrm{I} = \frac{\Sigma}{\Sigma_\lambda} = \frac{Q(T_\mathrm{c}^{-1} - T_\mathrm{h}^{-1})}{Q_\lambda(T_\mathrm{c}^{-1} - T_\mathrm{h}^{-1})} = \frac{Q}{Q_\lambda} = Nu \tag{1}$$

In thermally-induced natural convection without diffusion (i.e. in pure substances), the sources of irreversibility are heat diffusion by conduction and viscous friction. Non-dimensionalizing the corresponding entropy productions yields the respective numbers of irreversibility N_Iq and N_Iv, that must satisfy the following condition:

$$N_\mathrm{I} = N_\mathrm{Iq} + N_\mathrm{Iv} = Nu \tag{2}$$

Thermodynamic inconsistency in the usual Boussinesq equations

Combining the Gibbs equation ($T\mathrm{d}s = \mathrm{d}u^* + p\mathrm{d}v$), the entropy balance in its local form [$\rho\mathrm{D}s/\mathrm{D}t = -\nabla(T^{-1}\mathbf{q}) + \sigma$], and the UB assumptions, i.e. $\mathrm{d}v = = 0$ and ($\rho\mathrm{D}u^*/\mathrm{D}t = -\nabla\mathbf{q}$), results in $\boldsymbol{\sigma} = \mathbf{q}\nabla(T^{-1})$. In other words, the thermodynamic system described by the UB equations (and called "the UB system" in the following) recognizes only heat conduction (and not viscous friction) as a source of irreversibility. Indeed, any UB calculation leads to equality between N_Iq and Nu, where:

$$N_\mathrm{Iq} = \frac{1}{A_\mathrm{r}}\int_0^1\int_0^{x_\mathrm{c}} \frac{(\partial\theta/\partial x)^2 + (\partial\theta/\partial z)^2}{(1+\theta\Delta T/T_0)^2}\,\mathrm{d}x\,\mathrm{d}z, \quad \text{and}$$
$$Nu = -\frac{1}{A_\mathrm{r}}\int_0^1 \left(\frac{\partial\theta}{\partial x}\right)_{x=0}\mathrm{d}z \tag{3}$$

The UB system is thus in contradiction with equation (2). This contradiction with thermodynamics exists for two reasons. The first one, more visible but less significant, is the neglect in the heat equation of the heat generated by viscous friction, while the corresponding loss of kinetic energy is accounted for in the momentum equation. Thermodynamically speaking, in the UB system some kinetic energy (work) is lost in viscous friction but not transformed into heat. Remembering the words of Lavoisier: "Rien ne se perd" (nothing disappears), one deduces that that work lost in friction is necessarily released as work outside the system. Any exact transformation of work into work is a reversible process. This analysis shows that viscous friction does not create entropy in the UB system (when it does in the real world). Notice that losing kinetic energy is not sufficient for creating entropy in a convective system, irreversibility actually consists in the transformation of the lost work into heat, regardless of the magnitude of this heat rate compared to the other ones, conductive or advective.

Another feature shows that the UB systems cannot include viscous friction in its entropy balance: the number of viscous irreversibility is given by:

$$N_{\mathrm{Iv}} = \frac{1}{A_{\mathrm{r}}}\frac{\beta g H}{c_{\mathrm{p}}}\frac{T_0}{\Delta T}\int_0^1\int_0^{x_c}\frac{2(\partial u/\partial x)^2 + (\partial u/\partial z + \partial w/\partial x)^2 + 2(\partial w/\partial z)^2}{(1+\theta\Delta T/T_0)}\mathrm{d}x\,\mathrm{d}z, \quad (4)$$

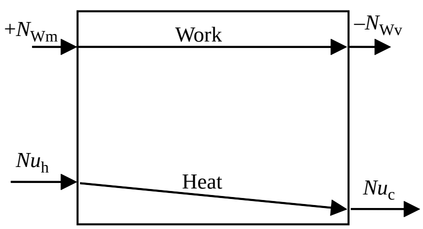
where appears a parameter, independent from A_{r}, Pr, and Ra, and which is absent from the UB problem: $\beta g H T_0/(c_{\mathrm{p}}\Delta T)$. How could a model lying on those three parameters only describe a phenomenon that involves a fourth one? Tritton (1988) noticed that that fourth parameter is the adiabatic temperature gradient ($\beta g T_0/c_{\mathrm{p}}$) non-dimensionalized in the problem framework (by $\Delta T/H$). As this fourth parameter is very often mentioned in the following it will be denoted by the symbol ϕ. Bejan (1984), and Gebhart *et al.* (1988) as well, mention that ϕ might easily be comparable to one, so that the viscous irreversibility "is not necessarily negligible" as written in both textbooks.

The second reason for inconsistency between the UB system and thermodynamics is that the usual heat equation does not contain any term involving transformation between heat (internal energy) and work within the fluid. This absence raises the question: where does the UB system find kinetic energy (work) for compensating the continuous viscous loss? The answer is: if not from the fluid itself, then from outside. In other words, the UB system somehow continuously receives mechanical energy from outside in order to compensate a continuous transfer of work (equating the viscous loss) to the outside. After this simple analysis, the energy transfers of the UB system in steady-state can schematically be represented as shown in Figure 1: in addition to the two equal heat fluxes exchanged with the heat sources, this system also exchanges two equal and opposite fluxes of mechanical energy with the surrounding, and there is absolutely no exchange between those two kinds of energy. Is it really natural convection?

As a consequence of this whole development the heat equation of the usual Boussinesq model (i.e. $\mathrm{D}T/\mathrm{D}t = \alpha\nabla^2 T$) should be questioned.

The thermodynamic Boussinesq model

Oberbeck's and Boussinesq's approach consisted in deriving approximations of the three basic transport equations (mass, momentum and energy) by neglecting terms of lower orders of magnitude. The same approach is widely used nowadays for building numerical models. About 46 years ago, Spiegel and Veronis (1960) developed



Notes: Energy diagram describing thermodynamic system defined by the usual Boussinesq equations in steady-state. In this UB system, which exchanges work with its surrounding, the fluxes of thermal energy (represented at the lower level) and of kinetic energy (upper level) are completely disconnected. $Nu_h = Nu_c$, and $N_{Wm} = N_{Wv}$, in steady-state

Figure 1.

a different approach: they considered the thermodynamic meaning rather than the orders of magnitude. They obtained what they called "the thermodynamic Boussinesq model" (denoted as TB in the following). This model can be called Boussinesq, because the temperature difference is still assumed small enough for neglecting the changes in fluid density and other thermophysical properties, except for the buoyancy term in the momentum equation. It is a thermodynamic model, because none of the processes, which are intrinsic to natural convection are discarded in the heat equation, especially the work of pressure stress and the heat generated by viscous friction. As a result, the heat equation in its enthalpic form is:

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \alpha \nabla^2 T + \frac{q_\mathrm{v}}{\rho c_\mathrm{p}} + \frac{T}{c_\mathrm{p}} \left(\frac{\partial (\rho^{-1})}{\partial T} \right)_\mathrm{P} \frac{\mathrm{D}P}{\mathrm{D}t} \tag{5}$$

The equation of state considered herein for the fluid is the usual one: $\rho = \rho_0[1 - \beta(T - T_0)]$. As the present study focuses on steady-states, only the hydrostatic pressure field is considered for the last term on the RHS, which thus finally transforms into: $-(\beta g/c_\mathrm{p})TV_z$. Lastly, taking the cavity height H, the speed $V^* = (\sqrt{Ra}\alpha/H)$, and ΔT as respective references for distances, velocity, and temperature difference $(T - T_0)$, the non-dimensional form of equation (5) is:

$$\frac{\mathrm{D}\theta}{\mathrm{D}\tau} = \frac{1}{Ra^{1/2}} \nabla^2 \theta + \frac{\beta g H}{c_\mathrm{p}} \left(\frac{\Phi}{Ra^{1/2}} - \theta w \right) - \phi w \tag{6}$$

Note that this equation does involve the parameter ϕ. The other involved parameter, βgH/c_p, is very small (of the order of 10^{-5}), while ϕ is "not necessarily negligible". In addition, when the development from the Gibbs equations to the entropy productions is now derived with equation (5) as heat equation, one correctly finds two causes of irreversibility: heat diffusion and viscous friction.

The boundary conditions considered herein are extremely common; fixed temperatures ($\pm$ 0.5) on the vertical walls, adiabatic horizontal walls, no slip on the four walls.

Numerical implementation

The modification of the heat equation is implemented into an initially usual Boussinesq model already described by Gadoin *et al.* (2001). The two complementary terms are treated like the other non-linear terms, i.e. implicitly through linear extrapolation. The extra CPU-cost of this modification is absolutely negligible. We investigate herein the square two-dimensional differentially heated cavity filled with air at 300K ($A_\mathrm{r} = 1$, $Pr = 0.71$). The grid is regular and staggered with a 256 × 256 mesh (the cases with $Ra > 10^8$ are calculated with 512 × 512). The configurations are stationary (Ra ranges from 3,000 to 10^8) and ϕ ranges from very small values ($\phi = 10^{-3}$, i.e. small cavities where the UB approximation is valid) to large ones ($\phi = 2$, for, which the largest value of H is 3.8 m).

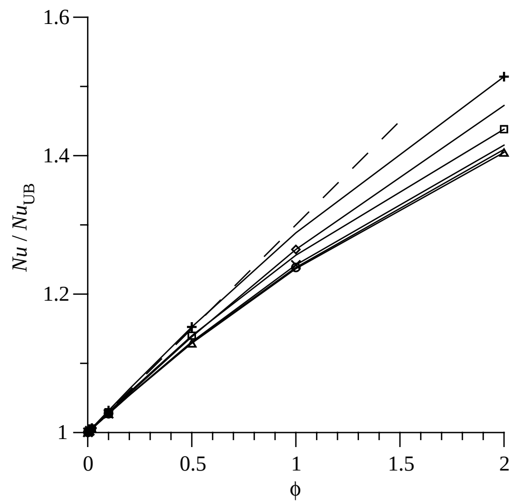
Thermodynamic balances

All the calculations done with the TB model yield $Nu_\mathrm{h} = Nu_\mathrm{c}$, and $N_\mathrm{Wm} = N_\mathrm{Wv}$ (the rates of work or heat are non-dimensionalized by the heat flux of the purely conductive system Q_λ). In addition, the second law balance now agrees with equation (2): full thermodynamic consistency is obtained.

The work of pressure forces and its effect

At given Ra number, the heat flux now depends on ϕ, i.e. on the cavity height. The dependence of Nu on ϕ for $3{,}000 \leq Ra \leq 10^8$ is shown in Figure 2 where the Nu number calculated with the UB model, Nu_{UB}, is taken as reference (Table I). It can be seen that the actual Nu number (calculated with the TB model) is always larger than Nu_{UB}, the difference can be quite significant when ϕ is of the order of unity. When ϕ is small compared to one, the relative difference is $0.3 \times \phi$ (in other words, if $\phi = 0.1$, the UB model underestimates the Nu number by 3 percent).

The term in DP/Dt in equation (5) describes the work exerted on the fluid by the hydrostatic pressure field. When non-dimensionalized into equation (6), the (by far) main part of that work is $-\phi w$, where w is the vertical component of velocity. When the fluid flows from regions of high pressure (bottom of the cavity) to regions of low pressure (top of the cavity), this term acts as a heat sink. In other words, when it flows upward close to the hot wall, the fluid generates work (just like in a turbine) and its internal energy is reduced by as much. On the other side of the cavity, close to the cold wall, the fluid flows downward, it receives work (just like in a compressor) and its internal energy increases: the term $-\phi w$ acts as a heat source. Globally, the combination of these two opposite exchanges between heat and work produces a heat transfer by a process, which operates



Notes: +: 3000; ◇ : 10^4; □ : 10^5; O: 10^6; △ : 10^7; ×: 10^8. The Nu numbers Nu_{UB} calculated with the usual Boussinesq model, i.e. with $\phi = 0$, are taken as reference (values are given in Table I). The dashed line corresponds to: $(Nu - Nu_{\mathrm{UB}})/Nu_{\mathrm{UB}} = 0.3 \times \phi$

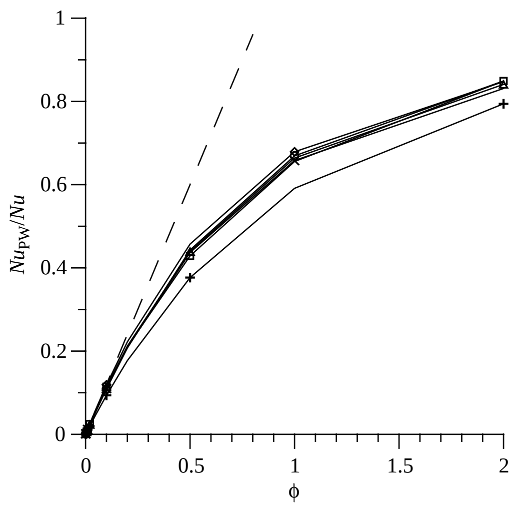
Figure 2. Dependence of the Nu number on the parameter ϕ for different Ra numbers

Ra	3,000	10^4	10^5	10^6	10^7	10^8
Nu	1.504	2.245	4.524	8.841	16.62	30.75

Note: Values of the Nu number calculated with $\phi = 0$ (usual Boussinesq model)

Table I.

in parallel to conduction plus advection. We will simply call this process the pressure-work (PW) effect. This process is similar to the piston effect identified by Onuki *et al.* (1990) and Zappoli *et al.* (1990), but somehow different. Indeed, the piston effect is due to fluid expansion and contraction, i.e. volume changes; when the fluid specific volume remains constant, the fluid internal energy changes like $c_v T$. The pressure-work effect is, in steady-state, due to motion in the hydrostatic pressure-field; when the fluid remains at constant pressure, the fluid specific enthalpy changes like $c_p T$. Actually, temperature-, volume- and pressure-changes all three co-exist in natural convection, and so do the piston and pressure-work (PW) effects. The distinction between them is not fundamental but it must be done when they are quantified. The proportion of the total heat transfer, which is due to the PW effect is obtained by subtracting from the total heat flux that due to conduction plus advection (with c_p as fluid heat-capacity) through the vertical mid-plane ($x = 0.5$). Non-dimensionalization yields a Nu number due to the PW effect, Nu_{PW}, and the ratio (Nu_{PW}/Nu) is the proportion of the Nu number, which is due to PW effect. This proportion is presented as a function of ϕ in Figure 3 for the same Ra numbers as in Figure 2. It can be noticed that as soon as convection is developed ($Ra \geq 10^4$), all the curves practically merge into a single correlation: ϕ is a pertinent parameter for describing this phenomenon. As expected, the PW effect is quite significant when ϕ is of the order of unity. Velarde and Perez-Cordon (1976) and Gray and Giorgini (1976) had already mention that the UB approximation requires ϕ to be small for being valid. Those authors recommend ϕ to be less than 0.1. However, the contribution of the PW effect to the total heat transfer in cavities has never been calculated before, even for small values of ϕ. Figure 3 shows that this contribution can be very significant and as high as $1.2 \times \phi$ when ϕ is small. In other words, when $\phi = 0.1$, the PW effect is responsible for 12 percent of the total heat transfer. Compared to the UB system, the work of pressure forces is also responsible for a strong reduction in convection intensity, so that the global changes in Nu number shown in Figure 2 actually result from the combination of a reduced convection with the PW effect.



Notes: The dashed line corresponds to: $Nu_{PW} / Nu = 1.2 \times \phi$

Figure 3.
Contribution of the pressure-work effect to the total heat transfer, as a function of ϕ and for different Ra numbers (same convention as in Figure 2)

From this analysis, the energy transfers occurring in natural convection (and accounted for in the TB equations) can be represented as shown in Figure 4. The exchanges between heat and work occurring within the fluid have a double result: first, a net production of kinetic energy (work) that compensates the loss in viscous friction; second, a heat transfer by the PW effect, which is larger by several orders of magnitude than the net produced work.

It must be here emphasized that all those results are obtained for steady-states and a fluid, which is far away from its critical point. The PW effect does not exist only in transient evolutions and in near-critical fluids. This statement also applies to the piston effect.

Validity domains for the different models

Thermodynamically speaking, there is no good reason for neglecting the work exerted by the hydrostatic pressure field on the flow and the viscous heat generation, because these are intrinsic components of buoyancy-induced natural convection. Nevertheless, validity of the UB approximation is a frequent issue. The validity limit stated by Gray and Giorgini (1976) or Velarde and Perez-Cordon (1976) was $\phi < 0.1$. The results presented in the previous section show that when $\phi = 0.1$, a process as large as 12 percent of the total heat flux is completely discarded by the UB model. It seems much more reasonable (especially when considering the huge progresses done in computing technologies during the last 30 years) to position the validity limit around $\phi = 0.01$ or 0.02.

Dimensionless quantities surely are fundamental. It is however interesting to consider the physical meaning of the different limits mentioned above, either between the Boussinesq and non-Boussinesq cases, or between the usual and thermodynamic Boussinesq cases. Figure 5 shows a diagram (ΔT, H), in log-log axes, established for air at 300K. Each value of Ra corresponds to a line with negative slope (solid lines); each value of ϕ corresponds to a line with positive slope (dashed lines). We have separated the whole domain in three regions. First, on the right, the region of large ΔT's, i.e. of non-Boussinesq configurations where Low-Mach-number models must be used.

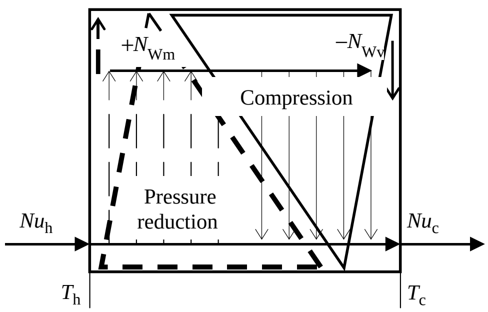


Figure 4.

Notes: Energy diagram describing natural convection and the thermodynamic Boussinesq system in steady-state (same conventions as in Figure 1). The large triangles symbolize the internal energy transformations between heat and work induced by the pressure forces

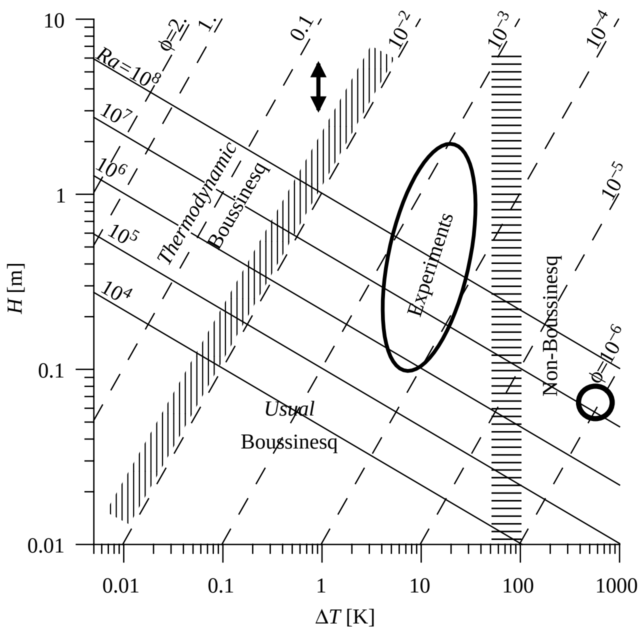


Figure 5.
Diagram (H, ΔT) showing the different domains of natural convection

Notes: The lines for prescribed values of Ra and ϕ are established for air at 300K. The region with large ΔT's (and – at fixed Ra – relatively small cavities and small ϕ's) is the one for *Non-Boussinesq* cases. The region with ϕ larger than 0.02 (and – at fixed Ra – relatively large cavities and small ΔT's) must be studied with the thermodynamic Boussinesq model. In between is the region of validity of the usual Boussinesq model

Second, in the upper left corner, the region of large ϕ's, i.e. of thermodynamic Boussinesq configurations where the TB model must be used. In between, lies the validity region of the usual Boussinesq model.

The open ellipse drawn in the latter region shows where are located typical experimental configurations: cavity size between 10 cm and 2 m; ΔT between 5 and 20K, i.e. large enough for being controlled (Salat *et al.*, 2004). It can be seen that the corresponding values of ϕ are very weak (10^{-4}-10^{-3}), which means that the PW effect is so small in experiments that it cannot be observed. It also results from this smallness that the UB model has always been well-adapted for comparing numerical calculations to experimental data; indeed, the agreement between the UB model and experiments is generally good. However, this does not prove that the UB model is universal.

The non-Boussinesq problems received some attention during the last years in literature. For instance, the open circle drawn in this region ($H \approx 7$ cm, $\Delta T \approx 700$K, $Ra \approx 10^7$) approximately shows a configuration simulated by Vierendeels *et al.* (2001). Compared to UB calculations, the non-Boussinesq effects (non-solenoidal flow, non-uniform viscosity) change the Nu number by 2-3 percent. What would be configurations where the pressure-work effect would modify the Nu number as much. The above results (Figure 2) show that such a change in Nu is obtained when ϕ is lies around 0.06-0.09. Keeping the same Ra number as above ($\approx 10^7$), these values of ϕ correspond to a ΔT of 0.1-0.15K applied to a 0.9-1 m high cavity. Yes, although the causes are very different, the configuration with a ΔT of 0.1K applied to a 1 m high enclosure departs from the usual Boussinesq case as much as a ΔT of 700K applied to a box of 7 cm. This simple example shows that some configurations looking very common however present a cavity height large enough for making the PW effect non negligible. Relatively high cavities can be found in architecture and housing engineering, so that configurations with a ΔT of 1K applied to a room of some meters (see the thick double-arrow in Figure 5) belong to the TB region. This fact, which is even more true for smaller ΔT's, received no attention until now.

Conclusions

The usual Boussinesq equations do not exactly represent buoyancy-induced natural convection. Indeed, the thermodynamic system actually simulated by those equations exchanges mechanical energy with its surrounding, so that it recognizes only heat diffusion as irreversibility. It results that the entropy balances obtained from UB models are not correct. Thermodynamic consistency is retrieved when both work of pressure forces and heat generated by viscous friction are accounted for in the heat equation: this is the thermodynamic Boussinesq model. Numerically, this correction is very simple and its cost in computing time is negligible. However, it re-introduces into the system a phenomenon, which is intrinsic to buoyancy-induced natural convection but unfortunately discarded by the usual Boussinesq equations: the exchanges between heat and work occurring within the fluid due to flow inside the hydrostatic pressure field. This pressure-work effect, which induces a heat-transfer in parallel to advection + conduction, is controlled by the adiabatic temperature gradient non-dimensionalized in the problem framework, $\phi = \beta g H T_0/(c_p \Delta T)$. This parameter ϕ is one of the control parameters of natural convection. Systematic numerical calculations done with the thermodynamic Boussinesq model show that the magnitude of the pressure-work effect can be as large as $1.2 \times \phi$, i.e. non negligible as soon as $\phi > 0.01$ or 0.02. Moreover, any theoretical study about buoyancy-induced natural convection (second law analyses, very probably stability analyses as well) should be done with the thermodynamic Boussinesq model.

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